

Piotr Pragacz – early years in Toruń

Andrzej Daszkiewicz, IMPANGA, 6.10.2023

Piotr was born in 1954 in Toruń. In 1969 he went to high school, II Liceum Ogólnokształcące, where his mathematics teacher was Zbigniew Staniszewski (1933-2023), renowned for his passion and talent. While at high school Piotr planned to become an architect and changed his plans only at the very last moment, towards the end of 4th grade, and decided to study mathematics instead. This decision might have been influenced by his teacher, but what was probably more important was another circumstance and other people Piotr met during his high school years.

At this time (and later as well), a group of mathematicians from the Institute of Mathematics of the Nicolaus Copernicus University in Toruń (UMK; founded in 1945, Copernicus did not attend it, but he was born in Toruń) led by professor Leon Jeśmanowicz, renowned for his work in math education, had been running a kind of Sunday School in Mathematics, for students of elementary (8th grade) and high schools from the Toruń area. Typically it was concentrated on problem solving (math olympiad type), but when Piotr started his 3rd grade, his class was taken over by a young mathematician, fresh after his PhD, Roman Kiełpiński (1939-1987). Instead of the usual problem solving, his idea was to show what mathematics really was, to show some of the main ideas of mathematics and their beauty. As the main book for the class he used Courant and Robbins' book "What is Mathematics", whose Polish translation was widely available at that time.

Inspired by these Sunday classes, some of the participants (Piotr Pragacz, Piotr Dowbor – now professor of mathematics at UMK, Wojciech Pieczyński – now professor of mathematical statistics in Paris, and Jerzy Weyman, currently professor at UJ in Kraków) started their own private meetings, at which they read some popular textbooks, including Kuratowski's "Differential and Integral Calculus". Pragacz, Dowbor and Weyman continued their private joint readings well into their studies (Pieczyński emigrated with his parents while still at high school).

At this time, in early 1970s, after the initial, "infant" period devoted mostly to abelian group theory, ring theory and some aspect of homological algebra, the Toruń algebra group, whose most prominent senior members had been Stanisław Balcerzyk and Edward Szaśiada (who subsequently moved to analysis), began to expand with wider interests of its younger members, fresh after doctorate, most notably Daniel Simson, Andrzej Tyc and Tadeusz Józefiak. Their interests included representations of algebras (Simson), Hopf algebras and invariant theory (Tyc) and commutative algebra and André-Quillen cohomology (Józefiak).

When Pragacz and Weyman entered the Toruń university in 1973 (Dowbor, a year younger, followed them in 1974 and joined the group led by Simson), they were advised by Kiełpiński to attend a new course on commutative rings given by Józefiak. In the next academic year, Igor Dolgachev, a student of Shafarevich, then from Moscow (since 1977 at Ann Arbor) came to Toruń for a longer visit and he gave a series of lectures on algebraic geometry which both Pragacz and Weyman attended. It was Piotr's first serious exposure to some of the ideas of algebraic geometry. At the third year of studies both Pragacz and Weyman once a month went to Warsaw to meet Maciej Skwarczyński, who guided them through complex variables theory, as no one with experience in this field was available in Toruń (they ended up reading Hörmander's several variables textbook and his solution of the "delta bar" problem). They spent the following year, their 4th year of studies, in Warsaw at Warsaw University, attending classes and seminars on algebra, algebraic geometry and topology.

At this time both were officially studying under the guidance of Józefiak (fresh after his habilitation), with the aim to concentrate on commutative and homological algebra and in particular on finite free resolutions that became a hot topic just a few years before, after some fundamental work of Eagon, Northcott, Hochster, Roberts, Buchsbaum and Eisenbud. One of the main problems was the construction of free resolutions of various classes of determinantal ideals. The case of maximal minors of a generic matrix had been solved by Eagon and Northcott, the case of submaximal minors of a generic symmetric matrix by Józefiak (and independently by Goto and Tachibana), and Piotr did it with Józefiak for submaximal pfaffians of a generic skew symmetric matrix, this became his master thesis in 1978.

In 1977 Alain Lascoux from Paris published his thesis, written under the guidance of Verdier and Schützenberger, where he used a geometric approach introduced by George Kempf in his 1971 Harvard thesis on Jacobian varieties of curves, together with representation theory of linear groups and combinatorics of symmetric functions, to construct modules of a free resolution in the case of minors of arbitrary fixed rank of a generic matrix. Invited by Józefiak, in 1978 Lascoux came to Toruń for the first of his many subsequent visits, where he met Piotr for the first time, and this meeting had a great influence on Piotr and in a sense shaped his future life as a mathematician.

The following few years were devoted by the whole group (Józefiak, Pragacz, Weyman and AD – the author joined the group in 1978) to the study of the work of Lascoux and others and filling many details apparently missing in the construction, most notably the maps between the modules. With the help of Józefiak, Weyman went to Brandeis University to work with Buchsbaum on his PhD related to this topic and Pragacz went to France for a longer visit with Dominique Foata in Strasbourg. Back in Toruń Piotr worked on his doctoral thesis on applying the Kempf-Lascoux technique to the case of ideals generated by pfaffians of skew-symmetric matrices (he finished his thesis in 1981, with

Józefiak as his advisor). When Weyman returned in 1980, they continued to work on the general theory of finite free resolutions for a few more years.

The culminating moment of this period of the activity of the group was the international conference "Young Tableaux and Schur Functors in Algebra and Geometry" that was organized in Toruń in the summer of 1980 by Józefiak and Lascoux, and whose results were then published as a volume 87-88 of *Astérisque*.

For a short time after his master thesis Piotr worked as an assistant in Institute of Mathematics at UMK in Toruń, but after his PhD he moved to the Toruń branch of IMPAN, where he stayed till late 90s, before moving to Warsaw.

In the early 80s, under the influence of Lascoux, Piotr became more and more interested in Schubert calculus, enumerative geometry and intersection theory. He worked with Józefiak on Schur Q-functions and with Witold Kraśkiewicz on the problem of realizing Schubert polynomials, defined by Lascoux and Schützenberger, as traces of certain representations of the group of invertible upper triangular matrices.

At the same time Piotr started to work on the theory of degeneracy loci of morphisms of vector bundles. His fundamental paper "Enumerative geometry of degeneracy loci" was published in 1988. The paper bears a dedication "To the memory of Roman Kiełpiński" (who died a year before). This paper became the core of Piotr's habilitation thesis.

Piotr gave several courses for students at UMK, e.g. on intersection theory and on characteristic classes. For one year he also ran a seminar on vector bundles on projective spaces, based on the book of Okonek, Schneider and Spindler.

Apart from mathematics, Piotr and his wife Maria were involved in other activities, most notably in the action to help families of political prisoners during the martial law of 1981-83, and also in the distribution of the forbidden "samizdat" literature.

(The author is grateful to Tadeusz Józefiak, Piotr Dowbor and Jerzy Weyman for providing some informations used above. All errors are due to author's failing memory!)